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Chiral extrapolation of the $X(3872)$ binding energy

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Abstract. The role of pion dynamics in the $X(3872)$ charmonium-like state is studied in the framework of a renormalisable effective quantum field theory approach and they are found to play a substantial role in the formation of the X. Chiral extrapolation from the physical point to unphysically large pion masses is performed and the results are confronted with the lattice predictions. The proposed approach overrides the gap between the lattice calculations and the physical limit in m_π.

1. Introduction

In spite of its relatively long history, the meson-like state $X(3872)$ in the spectrum of charmonium [1] remains not fully understood theoretically. According to the Particle Data Group, this state resides very close to the neutral $D\bar{D}^*$ threshold [2],

$$E_B = M_{D^0} + M_{\bar{D}^{*0}} - M_X = (0.12 \pm 0.26)\ \text{MeV}, \tag{1}$$

so it is natural to assume its wave function to have a large molecular admixture. The 1^{++} quantum numbers of the X determined recently by the LHCb Collaboration [3, 4] are consistent with its interpretation as an S-wave $D^0\bar{D}^{*0}$ bound state, however the binding mechanims in the X still remain an open question. In view of the fact that the 3-body $(D^0\bar{D}^0\pi^0)$ threshold resides below the 2-body $(D^0\bar{D}^{*0})$ threshold and the splitting between these thresholds is quite small, approximately 7 MeV, the role played by pions in the X should be important and the 3-body dynamics is expected to strongly affect the properties of the X. The small binding energy (1) allows for an effective field theory (EFT) formulation of the problem — see, for example, [5] for the pionless EFT based on pure contact $D\bar{D}^*$ interactions or [6, 7] for the so-called X-EFT approach with pions treated perturbatively. In [8] the properties of the $X(3872)$ molecular state were studied in a heavy-meson EFT framework with nonperturbative pions including all relevant 3-body scales, and the dependence of the X binding energy on the pion mass was studied in [9]. Another important issue related to the 3-body forces in the X is the problem of the nonperturbative renormalisation of the 3-body Faddeev-type equations describing the

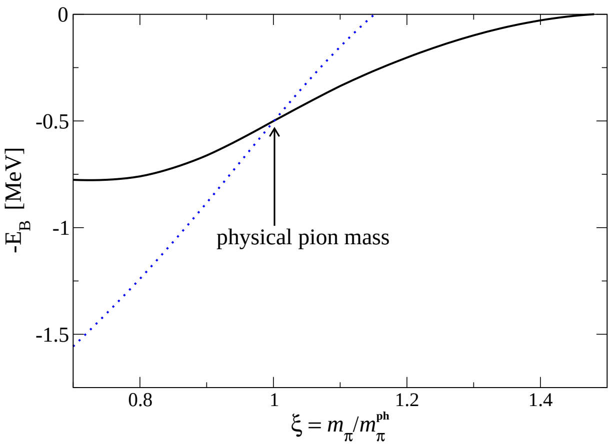


Figure 1. Pion mass dependence of the $X(3872)$ binding energy at LO. Solid line: the full calculation; dashed line: the static approximation.

interaction between heavy mesons in the X. The renormalisable approach proposed in [10] for the deuteron allows one to get rid of the finite cut-off artifacts in the leading order (LO) amplitude and to make a prediction for the m_π-dependence of the binding energy at this order, which then acquires NLO corrections. This approach can be applied to the $X(3872)$ that allows one to extrapolate the X binding energy to $m_\pi > m_\pi^{\rm ph}$ and therefore to establish a link between the theory and the lattice calculations traditionally performed for unphysically large pion masses [11, 12, 13].

2. Coupled-channel three-body equations

The key idea of the approach proposed in [10], that is the relativised formulation of the coupled channel equations, was generalised recently in [14] for the $D\bar{D}^*$ scattering, so the corresponding t-matrix elements $a_{00}^{nn'}(\boldsymbol{p},\boldsymbol{p}')$ and $a_{c0}^{nn'}(\boldsymbol{p},\boldsymbol{p}')$ in the C-even channel have the form

$$\left\{\begin{aligned} a_{00}^{nn'}(\boldsymbol{p},\boldsymbol{p}') &= \lambda_0 V_{00}^{nn'}(\boldsymbol{p},\boldsymbol{p}') - \sum_{i=0,c}\lambda_i\int d^3k V_{0i}^{nm}(\boldsymbol{p},\boldsymbol{k})\frac{1}{\Delta_i(\boldsymbol{k})}a_{i0}^{mn'}(\boldsymbol{k},\boldsymbol{p}'),\\ a_{c0}^{nn'}(\boldsymbol{p},\boldsymbol{p}') &= \lambda_c V_{c0}^{nn'}(\boldsymbol{p},\boldsymbol{p}') - \sum_{i=0,c}\lambda_i\int d^3k V_{ci}^{nm}(\boldsymbol{p},\boldsymbol{k})\frac{1}{\Delta_i(\boldsymbol{k})}a_{i0}^{mn'}(\boldsymbol{k},\boldsymbol{p}'),\end{aligned}\right. \tag{2}$$

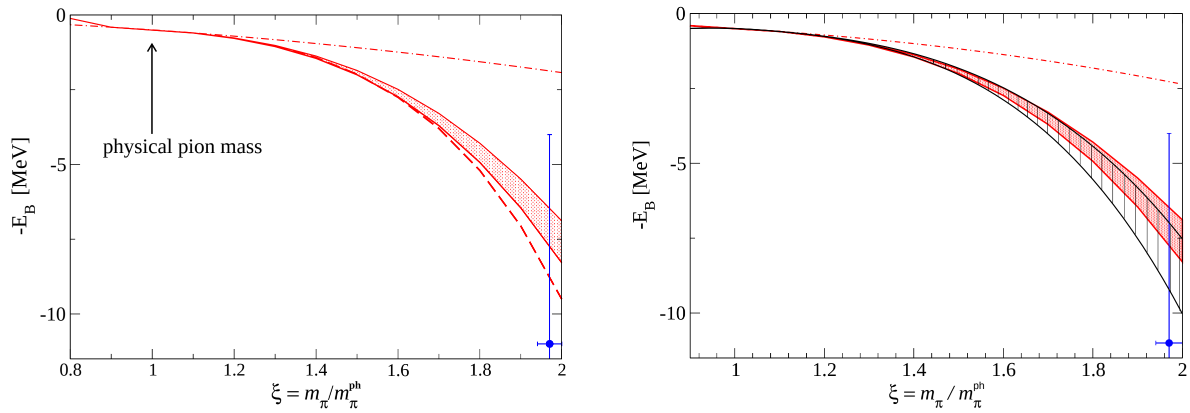
where $\lambda_0 = \langle 0|\vec{\tau}_1\cdot\vec{\tau}_2|\bar{0}\rangle = \langle c|\vec{\tau}_1\cdot\vec{\tau}_2|\bar{c}\rangle = 1$ and $\lambda_c = \langle 0|\vec{\tau}_1\cdot\vec{\tau}_2|\bar{c}\rangle = \langle c|\vec{\tau}_1\cdot\vec{\tau}_2|\bar{0}\rangle = 2$ are the isospin factors for the π^0- and $\pi^\pm$-exchange, respectively. The OPE potential $V_{ij}^{nn'}(\boldsymbol{p},\boldsymbol{p}')$ can be extracted from the lowest-order $D^*D\pi$ interaction Lagrangian taken in the form [7]

$$\mathcal{L} = \frac{g_c}{2f_\pi}\left(\boldsymbol{D}^{*\dagger}\cdot\boldsymbol{\nabla}\pi^a\tau^a D + D^\dagger\tau^a\boldsymbol{\nabla}\pi^a\cdot\boldsymbol{D}^*\right), \tag{3}$$

where the coupling constant g_c is related to the $D^{*0}\to D^0\pi^0$ decay width as

$$\varGamma(D^{*0}\to D^0\pi^0) = \frac{g_c^2 m_0 q^3}{24\pi f_\pi^2 m_{*0}}, \quad q = \lambda^{1/2}(m_{*0}^2, m_0^2, m_{\pi^0}^2)/(2m_*). \tag{4}$$

Here and in what follows, m_*, m, and m_π denote the masses of the D^* meson, D meson, and pion, respectively. Charged and neutral states are distinguished by an additional index, for example m_{*c} versus m_{*0}. Because of the P-wave nature of the $D^*\to D\pi$ decay the OPE



(a) (b)

Figure 2. Pion mass dependence of the $X(3872)$ binding energy at NLO. The result obtained for the m_π-dependent contact interaction only is shown with the dashed-dotted line.

contains a short-range part and is therefore well-defined only in connection with the short-range contact term C_0 [9],

$$V_{ij}^{nn'}(\boldsymbol{p},\boldsymbol{p}') = g_c^2(\boldsymbol{p}+\boldsymbol{p}')^n(\boldsymbol{p}+\boldsymbol{p}')^{n'}F_{ij}(\boldsymbol{p},\boldsymbol{p}') + C_0\,\delta^{nn'}, \tag{5}$$

where indices n and n' are contracted with the corresponding indices of the D^* polarisation vectors and $F_{ij}(\boldsymbol{p},\boldsymbol{p}')$ contains the relativised $D\bar{D}\pi$ and $D^*\bar{D}^*\pi$ propagators [14]. The energy E is counted from the neutral two-body threshold,

$$M = m_{*0} + m_0 + E. \tag{6}$$

3. Behaviour of the X binding energy for unphysical pion masses

At LO the contact interation is adjusted to reproduce the binding energy at the physical pion mass (for definiteness we set $E_B(m_\pi^{\rm ph}) = 0.5$ MeV), and the m_π-dependence of the E_B stems only from the pionic effects in the OPE potential and from those in the renormalised selfenergy loops. As a result, at LO of our EFT the dependence $E_B(m_\pi)$ can be predicted in a parameter-free way — see Fig. 1. So at LO the binding energy decreases with the pion mass growth and the 3-body effects are important for the dynamics of the system.

In NLO one includes corrections quadratic in m_π, also for the contact term. Notice that a stronger bound X for unphysically large m_π's is only possible if the short-range part of the interaction changes the slope of the binding energy at the physical point. Thus lattice predictions of a stronger bound X for $m_\pi > m_\pi^{\rm ph}$ indicate the importance of the short-range dynamics in the X. Since the nature of the short-range interaction is obscure, its dependence on m_π can only be guessed with the help of the ideas of naturalness. In particular, one can resort to the perturbative treatment of the m_π-dependent contact operator at NLO [14] (dashed line in figure 2(a)), use the heavy-meson formulation with the finite cut-off $\Lambda \in [500$ MeV, 700 MeV] [9] (black hatched band in figure 2(b)) or employ the nonperturbative resonance saturation approach [14] (red dotted band at both plots in figure 2). The dashed-dotted line in both plots in figure 2 corresponds to the calculation without pions. From figure 2 one can see a good agreement between different approaches to the naturalness. The predicted value of the binding energy at $m_\pi = 266$ MeV appears to be consistent with the lattice result from reference [13] shown as the blue dot with an error bar. As in LO, one can observe a strong influnce of the pion dynamics on the system.

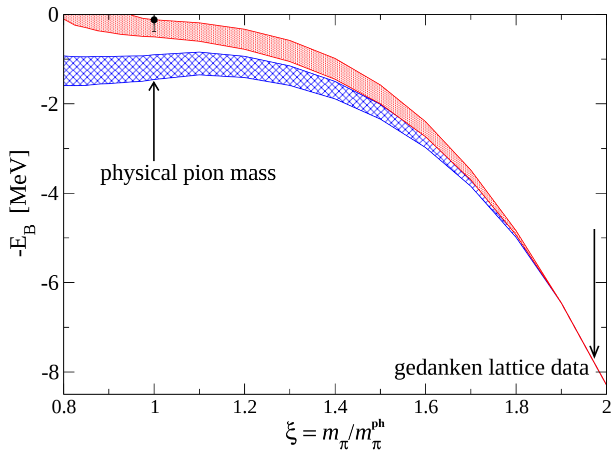


Figure 3. Pion mass dependence of the $X(3872)$ binding energy. The red dotted band is for the full calculation with dynamical pions at NLO while the blue crossed band is for the static OPE.

Alternatively one can start from the lattice results obtained for unphysically large pion masses and use them as an input to adjust the short-rang part of the potential. Then the binding energy can be extrapolated to the physical point and confronted with the experimental data. This approach is illistrated in figure 3 with the help of gedanken lattice data. The role of the 3-body dynamics is also clearly seen in this figure.

4. Conclusions

In conclusion we emphasise that our approach opens the way to override the gap between the unphysically large pion masses used on the lattices and the physical limit for the $X(3872)$ charmonium-like state. It can also be adapted to other near-threshold states, especially to those in which the three-body dynamics plays an important role.

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